Complex Map $z \to z + 1/z$

Look at the functions $z \to z^2$, $z \to 1/z$, $z \to z^2 + 2z$, $z \to e^z$ and their ATOs first.

This function is best applied to a Conformal Polar Grid. The image of the outside of the unit circle is the same as the image of the inside of the unit circle, namely the full plane minus the segment $[-2, 2]$. The unit circle is mapped to this intervall, each interior point $w = 2 \cdot \cos(\phi) \in [-2, 2]$ appears twice as image point, namely of $z = \exp(\pm i\phi)$.

The default choice shows how the outside of the unit disk is mapped to the outside of the interval $[-2, 2]$. If we note that $f'(\pm 1) = 0$ then we understand this behaviour: the interior $180°$ angle at these critical points ± 1 of the outside domain is again **doubled** to become the angle of the image domain (outside $[-2, 2]$) at ± 2.

A domain circle $z_R(\phi) = R\exp(i\phi)$ is mapped to the image ellipse $(R+1/R)\cos(\phi)+i(R-1/R)\sin(\phi)$, and a domain radius $z_\phi(R) = R\exp(i\phi)$ is mapped to the

Hyperbola $(R + 1/R)\cos(\phi) + i(R - 1/R)\sin(\phi)$, so the image grid therefore consists of a family of ellipses that intersect orthogonally a family of hyperbolae, and all these Conic Sections (see the Plane Curves Category) are “confocal”, i.e., they have the **same Focal Points**, namely at $+2$ and -2.

H.K.