About The Twisted Scherk Saddle Towers

The *Twisted Scherk Saddle Towers* are minimal surfaces that were found in 1988 as deformations of the Scherk Saddle Towers. Therefore one should first look at these latter simpler surfaces. One can imagine that one grips such a saddle tower at the top and the bottom and deforms the surface by twisting it. Of course it is not clear whether this can be done in such a way that the deformations stay minimal.

Fortunately, the most symmetric Scherk Saddle Towers carry straight lines through their saddles, and it is easy to imagine these lines staying on the twisted surface and remaining as lines of symmetry. Using these lines we can obtain the existence of the desired surfaces by solving the following Plateau Problem.

Consider a pair of adjacent half-lines through one saddle and another pair of half-lines, starting from the saddle above the first and with one of its half-lines vertically above the sector between the first two. These two "broken lines" cut a simply connected strip out of the surface. (One can see it by selecting "Don't Show Reflections" from the Action Menu) .

Now, vice versa, start with the two broken-lines and solve the Plateau problem for this infinite boundary to find a minimal strip they bound. Finally use 180° rotations around the half-lines to extend the Plateau strip to a complete minimal surface, a Twisted Scherk Saddle Tower. These surfaces played an important role in the development of the theory of minimal surfaces since—except for the Helicoid itself—they were the first examples having helicoidal ends,

The integer parameter ee controls the dihedral rotational symmetry of the surface: the angle of each pair of half lines above is π/ee. The parameter aa controls the amount of twist, with $aa = 0$ giving the straight Scherk Saddle Towers. We must keep $aa < \pi/ee$, since otherwise the existence construction fails. Of course the default morph varies aa.

In 3D-XplorMath minimal surfaces are computed via their Weierstraß representation. The Scherk Saddle Towers are parametrized by punctured spheres and our twist deformation does not change this conformal type. However, due to this twist, the Gauß map of the surface is not single-valued: it is a rather a multivalued function on the punctured sphere, and this makes the computation more difficult than for the other spherical minimal surfaces, since during the integration of the multivalued Weierstraß integrand, it is necessary to use analytic continuation in order to guarantee that we always have the correct value of the Gauß map.

The Weierstraß representation is given in:

Karcher, H., Embedded Minimal Surfaces derived from Scherk's Examples, Manuscripta math. 62 (1988), pp. 83 - 114.

Recently Traizet and Weber have found a new construction of embedded singly periodic minimal surfaces that can be illustrated with the twisted Scherk saddle towers. Choose aa close to its theoretical limit $1/ee$ and try to see the resulting surfaces as made out of ee ordinary helicoids. Obviously one has to allow modifications of the helicoids in the middle, but away from the middle one can see these helicoids well. Traizet and Weber were able to turn this observation around. They found conditions how to place a collection of helicoids so that they could prove the existence of a family of singly periodic embedded minimal surfaces which converged to the given helicoids in the same sense as the twisted Scherk saddle towers converge as $aa \to \pm 1/ee$.
H.K.