About the Symmetric 4-Noid

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The symmetric and skew 4-noids are parametrized by 4-punctured spheres; we use lines which extend polar coordinates around the punctures. Formulas are from [K2].

The intersection of the two families is the 4-noid from the Jorge-Meeks family of k-noids. These k-noids are the first finite total curvature immersions where the Weierstrass data were manufactured to fit a previously conceived qualitative global picture of the surfaces.

In these examples aa controls the angle between the ends, and should be kept in the range $0 < aa < 0.9$, (aa=0 gives a symmetric 4-noid). In the symmetric case, bb determinses the size of the catenoid ends.

We have added the corresponding surfaces of higher dihedral symmetry, $ee = 2$ (the default), 3, 4,... , and the corresponding default morphs. This gives surfaces with $2 * ee$ catenoid ends. Since the most beautiful surface among the k-noids is the 3-noid, we have also added the surfaces with an odd number of ends. In these cases there is no morph which changes the relative size of neighboring ends, i.e., the value of aa is irrelevant. These surfaces can be chosen with ee = 1.5 (Trinoid), 2.5, 3.5,... .

Our default suggestion is to morph the relative size of the opposite pairs of catenoid ends in the symmetric case and the angle between the catenoid ends in the skew case. The skew surface family goes from the Jorge-Meeks 4-noid to surfaces which look like two catenoids joined by a handle. This convinced David Hoffman that the idea of adding handles might be promising.

[K2] H. Karcher, Construction of minimal surfaces, in "Surveys in Geometry", Univ. of Tokyo, 1989, and Lecture Notes No. 12, SFB 256, Bonn, 1989, pp. 1–96.

For a discussion of techniques for creating minimal surfaces with various qualitative features by appropriate choices of Weierstrass data, see either [KWH], or pages 192–217 of [DHKW].

[KWH] H. Karcher, F. Wei, and D. Hoffman, The genus one helicoid, and the minimal surfaces that led to its discovery, in "Global Analysis in Modern Mathematics, A Symposium in Honor of Richard Palais' Sixtieth Birthday", K. Uhlenbeck Editor, Publish or Perish Press, 1993

[DHKW] U. Dierkes, S. Hildebrand, A. Kuster, and O. Wohlrab, Minimal Surfaces I, Grundlehren der math. Wiss. v. 295 Springer-Verlag, 1991