About the Space Curves Category

A space curve is described by a map of an interval, $[tMin, tMax]$ in t-space into (x, y)-space. Each point t is mapped to a three-dimensional point P with coordinates $(x(t), y(t), z(t))$. The mapping is given by three real-valued functions that give the values of $x(t)$, $y(t)$, and $z(t)$ as a function of the t and nine global "parameters", called $aa, bb, cc, \ldots, ii$. Most curves use only 1,2,3 or even none of these parameters. To discretize the curve, the interval $[tMin, tMax]$ is subdivided into $tResolution$ equi-spaced points running from $tMin$ to $tMax$. These points divide the domain interval into subintervals. When a curve is "Create"-ed, the mapping $t \mapsto (x, y, z)$ is applied to each of the subdivision points, so each of the subintervals in t-space is mapped to a segment in 3-space, and this collection of segments is a polygon that is what actually represents the immersed curve.

When you have selected a particular curve from the Space Curve menu, a version with certain default parameter values will be displayed. You can then choose "About This Object..." from the Action Menu to see the equations for the curve as a function of t and the parameters, (and perhaps to see some interesting properties of the curve). You can change the parameters in the Settings Menu and then re-Create the curve.

The program can also interpolate linearly between two curves of the same family that you can set by choosing "Set Morphing..." in the Settings Menu. The number of steps in the "morph" is the Number of Frames in the filmstrip, an integer that you can also set. Playing back the filmstrip gives a "movie" of the curve changing gradually ("morphing") between the initial and final curves.

A user can define a curve by choosing one of the User Defined... items from the Space Curve menu. This will bring up a dialog that will permit one to create algebraic expressions describing the curve involving the variables t and the nine parameters, $aa, bb, \ldots, ii$.

You can also define a space curve by giving its curvature, κ, and its torsion, τ, as a function of t (and the usual parameters $aa, \ldots, ii$). To do this choose User (Curvature & Torsion)... from the Space Curves menu. The resulting curve will have t as its arclength parameter, and will start (at $t = 0$) from the origin with tangent the unit vector in the x direction. (By the fundamental theorem of space curves, there is a unique such curve.) The interval $[tMin, tMax]$ must contain zero of course—or the curve will be empty.

See the corresponding discussion in About the Surface Category for more detail on how to enter expressions.

Space curves can also be given "implicitly", as the solutions of two equation, $f(x, y, z) = ff$ and $g(x, y, z) = gg$. There is a User (Implicit)... menu item in the Space Curve menu that brings up a dialog that will let you enter expressions (defining the function $f(x, y, z)$) and $g(x, y, z)$ and values for ff and gg, and a rectanglular box in the x, y, z-space. Clicking on the Create button will display the simultaneous solution of $f(x, y, z) = ff$ and $g(x, y, z) = gg$ inside the box.

The Action Curve menu has an item "Show as Tube". If you select it, the current space curve will be drawn as a "tube", i.e., instead of the curve itself being drawn, a surface that is the boundary of a tube with polygonal cross-section centered on the curve gets drawn. The cross-section of the tube is a regular n-gon, where $n = 4$ by default, but you can choose any value between 3 and 18 in the Set Resolution And Scale dialog. To draw the tube it is necessary to choose a frame field along the curve. There are two natural choices—the Frenet frame field and a parallel frame field. The Frenet frame is chosen by default, but you can switch between them using the Action menu.

A good way to see the Frenet "moving trihedral" (or the parallel one) is to choose "Show Repère Mobile" from the Action menu. This is particularly striking in stereo! Another interesting animation is provided by showing projections of a small part of the curve on the osculating, normal, and rectifying planes (or on all three!) at a point that moves along the curve. Again, this is best seen in stereo.