Functions with compact levels in 3D-XplorMath

One should always experiment with the level value v of the function f. In 3DXM: $v = ff$. For small values of ff one will see how the function was designed by guessing the degenerate level $f = 0$. The `Default Morph` often varies ff, for example showing non-singular levels converging to the singular one. In some cases other parameters are morphed, for example to get larger values of the genus g. Some cases offer: `Flow to Minimum Set` $\{f = 0\}$ (see Action Menu). (Artificial looking denominators in the following prevent the function f from growing too fast.)
Note that the Action Menu has many decorations for implicit surfaces: Curvature line fields, net of curvature lines, normal curvature circles, geodesics with mouse chosen initial data, geodesic nets.

Pretzel: See page 5 of *Explicit versus Implicit Surfaces.* The surface has genus 0,1,2 or 3, depending on parameter values.
$f(x, y, z) := h(x, y, z)^2 + (1 + cc)z^2$ with

$$h(x, y, z) := \frac{((x - cc)^2 + y^2 - 1) \cdot ((x + cc)^2 + y^2 - 1)))}{1 + (1 + cc)(x^2 + y^2)}$$

Bretzel2, a genus 2 tube around a figure 8, genus 0 for large ff:

$$f(x, y, z) := \frac{(((1 - x^2)x^2 - y^2)^2 + z^2/2)}{(1 + bb(x^2 + y^2 + z^2))}.$$

Bretzel5, a genus 5 tube around two intersecting ellipses:
$f(x, y, z) := ((x^2 + y^2/4 - 1) \cdot (x^2/4 + y^2 - 1))^2 + z^2/2.$

Pilz, a genus 3 tube around circle and orthogonal ellipse:
$f(x,y,z) :=$
$((x^2+y^2-1)^2+(z-0.5)^2)\cdot(y^2/aa^2+(z+cc)^2-1)^2+x^2)$
$-\,dd^2(1+bb(z-0.5)^2)$.
`Default Morph:`$0.03 \le cc \le 0.83$.

Orthocircles, a genus 5 tube around three intersecting orthogonal circles ($aa = 1,\ ff = 0.05$) or a tube around three Borromean ellipses ($aa = 2.3, ff = 0.2$) – choose in the Action Menu.
$f(x,y,z) :=$
$((x^2/aa+y^2-1)^2+z^2)\cdot((y^2/aa+z^2-1)^2+x^2)\cdot$
$((z^2/aa+x^2-1)^2+y^2)$.
Use: `Flow to Minimum Set` $\{f=0\}$ (from Action Menu).

DecoCube, tube around six circles of radius cc on the faces of a cube. Genus 5,13,17, depending on cc, ff:
$f(x,y,z) := ((x^2+y^2-cc^2)^2+(z^2-1)^2)\cdot$
$((y^2+z^2-cc^2)^2+(x^2-1)^2)\cdot((z^2+x^2-cc^2)^2+(y^2-1)^2)$.
`Default Morph:` $ff = 0.02,\ 0.25 \le cc \le 1.3$.
Use: `Flow to Minimum Set` $\{f=0\}$ (from Action Menu).

DecoTetrahedron has as its minimum set four circles on the faces of a tetrahedron. The formula is similar but more complicated than the previous one. cc changes the radius of the circles, bb changes their distance from the origin, ff selects the level. Use: `Flow to Minimum Set` to see the circles used for the current image.
The `Default Morph` changes cc and with it the genus.

JoinTwoTori is a genus 2 surface such that the connection between the two tori does not much distort them if ff is small. (It is used for genus-2-knots in Space Curves.)

$$
\begin{aligned}
Tor_{right} &:= ((x-cc)^2 + y^2 + z^2 - aa^2 - bb^2)^2 \\
&\qquad + 4aa^2(z^2 - bb^2) \\
Tor_{left} &:= ((x+cc)^2 + y^2 + z^2 - aa^2 - bb^2)^2 \\
&\qquad + 4aa^2(z^2 - bb^2)
\end{aligned}
$$

$$
f(x,y,z) := \frac{Tor_{right} \cdot Tor_{left}}{1 + (x-cc)^2 + (x+cc)^2 + y^2 + z^2/2}.
$$

The `Default Morph:` $0.01 \le ff \le 2.5$ joins the tori.

CubeOctahedron

The level surfaces of the function
$f_{cube}(x,y,z) := \max(|x|,|y|,|z|)$ are cubes.
The level surfaces of the function
$f_{octa}(x,y,z) := |x| + |y| + |z|$ are octahedra.

$\tilde{a} := \min(2 \cdot aa, 1)$, $\tilde{b} := 2 \cdot \min(bb, 1)$. These coefficients for the following linear combination allow an interesting morph.

$f(x,y,z) := \max(\tilde{a} \cdot f_{octa}(x,y,z) + \tilde{b} \cdot f_{cube}(x,y,z))$.

Default: $aa = 0.5, bb = 1, ff = 1$. This truncated cube is Archimedes' Cubeoctahedron.
`Default Morph:` $aa = \frac{2}{3} \to \frac{1}{3}$, $bb = 0.5 \to 1.5$, $ff = 1$.
This deformation from the octahedron to the cube passes through three Archimedean solids.