Chain of Half-Catenoids and Field of Half-Catenoids *

These surfaces played no role in the history of minimal surfaces, but they explain very nicely how the Weierstrass representation works, compare Half-Catenoids and Weierstrass Representation.

Weierstrass representation of a vertical catenoid:

$$g(z) = z,\ dh = \frac{dz}{z}.$$

Weierstrass representation of a singly periodic chain of vertical half-catenoids:

$$g(z) = bb \cdot \sin(z),\ dh = \frac{dz}{\sin(z)}.$$

The simple zeros of the sine function create half-catenoid punctures. These punctures have no real periods since two orthogonal planes of mirror symmetry cut each half-catenoid into four congruent pieces. Neighboring half-catenoids are separated by straight lines. They are axes of 180° rotation symmetry. One needs to compute the surface only in a strip between two neighboring lines. The scaling

* This file is from the 3D-XplorMath project. Please see: http://3D-XplorMath.org/

TOC

factor bb in the Gauss map controls the size of the half-catenoids relative to the distance between them. Two such strips are a fundamental domain for the group of translation symmetries.

Weierstrass representation of a doubly periodic field of vertical half-catenoids:

$$g(z) = bb \cdot J_F(z),\ \ dh = \frac{dz}{J_F(z)}.$$

The function J_F is a doubly periodic function on $\mathbb{C}$, in this case with a rectangular fundamental domain. In each fundamental domain J_F has two simple zeros and two simple poles, see 'Symmetries of '`Elliptic Functions`''. The zeros of the Gauss map g together with the poles of dh create the half-catenoid punctures. The poles of g are cancelled by the zeros of dh, they give the polar centers on the surface. As in the previous examples we have orthogonal planes of mirror symmetry cutting each half-catenoid and we have straight lines on the surface, running between the half-catenoids. These symmetries of the minimal surface are a consequence of the corresponding symmetries of the elliptic function which determines the Weierstrass data. The scaling parameter bb of the Gauss map changes the size of the half-catenoids relative to their distance.

H.K.