Complex Map $z \mapsto \sqrt{z}$

Look at the functions $z \mapsto z^2$, $z \mapsto 1/z$ and their ATOs first.

The function $z \mapsto \sqrt{z}$ should be looked at using both Cartesian and Polar Grids

Note that since this function is the inverse of $z \to z^2$, we expect to see related phenomena: circles around 0 go to circles around 0, radial lines from 0 go to radial lines from 0, but now with **half** the angle between them (since we look at the inverse map). A neighbourhood of 0 was very much contracted by $z \to z^2$, now we see the opposite, the distance of points from zero is increased very much (beyond any Lipschitz bound).
A more complicated aspect is the fact, since all $z \neq 0$ have two distinct square roots differing by a factor of -1, the function $z \mapsto \sqrt{z}$ is not really a well defined map until we make some choices.

The function $\sqrt{z}$ used by 3D-Filmstrip maps the upper half plane to the first quadrant, the (strict) lower half plane to the fourth quadrant, and the negative

real axis to the positive imaginary axis—so there is no continuity from above to below the negative real axis (which is therefore called a "branch cut").

The Cartesian grid lines are mapped to two families of **hyperbolae** which intersect each other orthogonally.

H.K.