Complex Map $z \mapsto 1/z$

(REMARK: The actual mapping for this example is $z \mapsto aa/(z - bb) + cc$, with the default values $aa = 1$, $bb = 0$, and $cc = 0$.)

Look at the function $z \mapsto z^2$ and its ATO first.

The function $z \mapsto 1/z$ should be looked at both in Cartesian and Polar Grids.

Notice first:

1) The real axis, imaginary axis and unit circle are mapped into themselves,
2) the upper half plane and the lower half plane are interchanged, and
3) the inside of the unit circle and its outside are also interchanged.

This is best seen in the (default) Conformal Polar Grid. In the Cartesian Grid one should in particular observe that all straight parameter lines (in the domain) are mapped to circles (some exceptions, like the real axis, remain lines). The behaviour of these circles near zero can be looked at as an image of the behaviour of the standard Cartesian Grid near infinity. In fact **all** circles are mapped to circles or lines.

Examples to look at after this are

$$z \mapsto (az+b)/(cz+d) \text{ and } z \mapsto (z+cc)/(1+\bar{c}cz),$$

both of which can be obtained from $z \to 1/z$ by composition with translations $z \to z+a$ or scaled rotations $z \to a \cdot z$. Therefore all of these so-called "Möbius transformations" map circles and lines to circles and lines.

H.K.