The Complex Exponential Map $z \mapsto e^z$

(REMARK: The actual mapping for this example is $z \mapsto \exp(aa(z - bb)) + cc$, with the default values $aa = 1$, $bb = 0$, $cc = 0$.)

Look at the functions $z \mapsto z^2$, $z \mapsto 1/z$ and their ATOs first.

The complex exponential function $z \mapsto e^z$ is one of the most marvellous functions around. It shares with the real function $x \mapsto \exp(x)$ the differential equation $\exp' = \exp$ and the functional equation $\exp(z+w) = \exp(z) \cdot \exp(w)$.

This latter identity implies that one can understand the complex Exponential in terms of real functions, for if we put $z = x + i \cdot y$ then we have

$$\exp(x + i \cdot y) = \exp(x) \cdot \exp(i \cdot y) =$$

$$\exp(x) \cdot \cos(y) + i \cdot \exp(x) \cdot \sin(y).$$

This says that a Cartesian Grid is mapped "conformally" (i.e., preserving angles) to a Polar Grid: the parallels to the real axis are mapped to radial lines, and segments of length 2π that are parallel to the imaginary axis are mapped to circles around 0. This function is therefore used to make, in the Action Menu, the Conformal Polar Grid. Observe how justified it is to describe the image grid as "made out of curved small squares".

If you have seen $z \mapsto e^z$ and $z \mapsto z + 1/z$ then now look at $z \to \sin(z)$.

H.K.