The Quadratic Henon Map and its Attractor*

This Henon Map visualization gives the orbit under iteration of the map $(x, y) \to (y + 1 - aa\ x^2, bb\ x)$.

The default values are $aa = 1.4$ and $bb = 0.3$. The initial point is $(x, y) = (cc, dd)$, defaults $cc = 1.0, dd = 1.0$. The number of iterations plotted is ee, but the first ff iterates are omitted. The defaults are $ee = 3000$ and $ff = 20$. The map has two fixed points, they are obtained by solving a quadratic equation and they are marked by small circles. Also from a quadratic equation one obtains an orbit of period 2, marked by small spades. Numerically we found no indication of an orbit of period 3. We found one orbit of period 4 numerically and marked it by small squares. — The derivative of the Henon map has determinant $-bb$, i.e. the Henon map reduces area with a uniform rate. One can view the eigen directions of the derivative via an Action Menu entry. — The inverse of the Henon map is the quadratic map $(X, Y) \to (\frac{1}{bb}Y, X - 1 + \frac{aa}{bb^2}Y^2)$.

In 3DXM, to move the finished image, drag the image with the mouse. To zoom in our out, drag vertically with the Shift key pressed. (If you zoom in, you might want to increase parameter ee using Settings > Set Parameters.) To zoom into a particular region, hold down Command and then drag a rectangle in the window, then the program will

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* This file is from the 3D-XplorMath project. Please see: http://3D-XplorMath.org/

zoom into that region of the Henon attractor, allowing you to see it in greater detail.

Are there more periodic points on the Henon attractor? An Action Menu entry allows to search for orbits of period ii. Since numerical inaccuracies, e.g. of the period 2 orbit, build up after about 60 iterations to visibly leave the orbit, we restricted to: $ii < 80$. Each numerically found orbit is iterated until the user stops the iteration. If this stop is by mouse button, the orbit is saved and the last orbit of these is added to the visually indicated periodic points.

The Henon map depends strongly on the parameters. For example, we can obtain an attracting orbit of period 6 with $aa = 1.45$. To see it, change aa, stop the iteration and click in the Action Menu: `Do 500 Iterations`.
(Morphing aa and bb works, but there is no default morph, so first select Set Morphing... from the Settings menu to set up the morph—be sure to click the Init To Current Values button, then change aa0 aa1, bb0 and bb1.)

Finally, we added to the Action Menu: `Use Hit Count Coloration`. This changes the representation by adding vertical bars over the points of the Henon attractor. The length of these bars shows how often that - pixelsized - point of the attractor is visited during the iteration. The graphic therefore illustrates the iteration-invariant measure on the attractor. For a 1-dimensional such hit count see the Feigenbaum Tree.

H.K.