About The Dragon Curve*

see also: Koch Snowflake, Hilbert SquareFillCurve
To speed up demos, press DELETE

The Dragon is constructed as a limit of polygonal approximations D_n. These are emphasized in the 3DXM default demo and can be described as follows:
1) D_1 is just a horizontal line segment.
2) D_{n+1} is obtained from D_n as follows:

a) Translate D_n, moving its end point to the origin.

b) Multiply the translated copy by $\sqrt{1/2}$.

c) Rotate the result of b) by $-45°$ degrees and call the result C_n.

d) Rotate C_n by $-90°$ degrees and join this rotated copy to the end of C_n to get D_{n+1}.

The fact that the **limit points** of a sequence of longer and longer polygons can form a two-dimensional set is not really very surprising. What makes the Dragon spectacular is that it is in fact a **continuous curve** whose image has positive area—properties that it shares with Hilbert's square filling curve.

There is a second construction of the Dragon that makes it easier to view the limit as a curve. Select in the Action Menu: *Show With Previous Iteration.*

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* This file is from the 3D-XplorMath project. Please see: http://3D-XplorMath.org/

This demo shows a local construction of the Dragon: We obtain the next iteration D_{n+1} if we modify each segment of D_n by replacing it by an isocele 90° triangle, alternatingly one to the left of the segment, and the next to the right of the next segment. This description has two advantages:
(i) Every vertex of D_n is already a point on the limit curve. Therefore one gets a dense set of points, $c(j/2^n)$, on the limit curve c.
(ii) One can modify the construction by decreasing the height of the modifying triangles from $aa = 0.5$ to $aa = 0$. The polygonal curves are, for $aa < 0.5$, polygons without self-intersections. This makes it easier to imagine the limit as a curve. In fact, the *Default Morph* shows a deformation from a segment through continuous curves to the Dragon—more precisely, it shows the results of the $(ee = 11)$th iterations towards those continuous limit curves.

As another option, one can vary the parameter bb through integer values $bb = 2, 3, \ldots$ to obtain other families of Fractal curves (from the Action Menu only).

Finally, one can choose in the Action Menu to map any selected Fractal curve by either the complex map $z \to z^2$ or by the complex exponential. The program waits for a mouse click and then chooses the mouse point as origin.

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