Complex Map $z \to z^2$

(REMARK: The actual mapping for this example is $z \mapsto aa(z - bb)^{ee} + cc$, with the default values $aa = 1$, $bb = 0$, $cc = 0$, and $ee = 2$.)

Look at the discussion in "About this Category" for what to look at, what to expect, and what to do.

Just as the appearance of the graph of a real-valued function $x \mapsto f(x)$ is dominated by the critical points of f, it is an important fact that so also, for a conformal map, $z \mapsto f(z)$, the overall appearance of an image grid is very much dominated by those points z where the derivative f' vanishes. Most obviously, near points a with $f'(a) = 0$ the grid meshes get very small and, as a consequence, the grid lines usually are strongly curved. If one looks more closely then one notices that the angle between curves through a is **not** the same as the angle between the image curves through $f(a)$ (recall: $f'(a) = 0$). We will find this general description applicable to many examples.

One should first look at the behaviour of the simple quadratic function $z \to z^2$ near $a = 0$, both in Cartesian and in Polar coordinates. One sees that a rectangle, which touches $a = 0$ from one side is folded around 0 with strongly curved parameter lines, and one also sees in Polar coordinates that the angle between rays from 0 gets **doubled**. The image grid in the Cartesian case consists of two families of orthogonal intersecting parabolas.
One should return to this prototype picture after one has seen others like $z \to z+1/z$, $z \to z^2+2z$ and even the Elliptic functions and looked at the behaviour near their critical points.

The first examples to look at, (using Cartesian **and** Polar Grids) are $z \to z^2,\ z \to 1/z,\ z \to \sqrt{z},\ z \to e^z$.
H.K.