Complex Map $z \to \sin(z)$

Look at the functions $z \to z^2$, $z \to 1/z$, $z \to z^2 + 2z$, $z \to e^z$ and their ATOs first.

While the behaviour of the one-dimensional real functions $x \mapsto \exp(x)$ and $x \mapsto \sin(x)$ are quite dissimilar (exp is convex and positive, while sin is periodic and bounded), as complex functions they are very closely related:

$$\sin(z) = \frac{\exp(iz) - \exp(-iz)}{2i},$$

an identity that explains why the image grid under sin of the default Cartesian grid looks exactly like the image grid under $z \to z + 1/z$ applied to a Conformal Polar Grid outside the unit circle. For if we put $w(z) := \exp(iz)/i$, then $\sin(z) = (w(z) + 1/w(z)))/2$, and recall that exp maps the standard Cartesian Grid to the Conformal Polar Grid around 0. The parameter curves in the image grid of sin are therefore the same orthogonal and confocal ellipses and hyperbola as in the image of $z \mapsto z + 1/z$.

H.K.