The Fractional Linear Maps

$$z \mapsto (a \cdot z + b)/(c \cdot z + d)$$

Look at the functions $z \mapsto z^2$, $z \to 1/z$ and their ATOs first.

These functions are called or fractional linear maps or **Möbius transformations**. They differ from the map $z \mapsto 1/z$ by composition with a translation $z \mapsto z + a$ or scaled rotations $z \mapsto a \cdot z$. As discussed for $z \mapsto 1/z$ they transform lines and circles to lines and circles.

The default special case is $z \mapsto (z - 1)/(z + 1)$. It is best understood in the (default) Conformal Polar Grid. Since it maps 0 to -1 and ∞ to $+1$, one can see the Polar coordinate centers moved from $0, \infty$ to $-1, +1$. This picture is the first step towards understanding the complex (or "Gaussian") plane plus the point at infinity as the "Riemann Sphere".

See also the other Möbius transformations from the Conformal Maps menu.

H.K.