The Möbius Transformation

$$z \to \frac{(z+cc)}{(1+\overline{cc}\cdot z)}.$$

of the unit disk.

Look at the Möbius transformation

$$z \to \frac{(a\cdot z+b)}{(c\cdot z+d)}$$

and its ATO first.

This function maps the interior of the unit disk bijectively to itself, for every choice of cc with $|cc|<1$. The behaviour outside of the unit disk is obtained by reflection in the unit circle, i.e., $z \to 1/\bar{z}$.

These maps have an interesting geometric interpretation: they are isometries for the "hyperbolic metric" on the unit disk. To understand this further, imagine that the unit disk is a map of this two-dimensional hyperbolic world and that the scale of this map is not a constant but equals $1/(1-z\bar{z})$. This means that we do not obtain the length of a curve $t \to z(t)$ as in the Euclidean plane by the integral $\int |z'(t)| dt$—we have to take the scale into account and define its hyperbolic length by $\int |z'(t)|/(1 - |z(t)|^2) dt$. It is this hyperbolic length of curves that is left invariant by the "hyperbolic translations" $z \to (z + cc)/(1 + \overline{cc} \cdot z)$. Locally the Pseudosphere (Category: Surfaces) has the same hyperbolic geometry.

H.K.