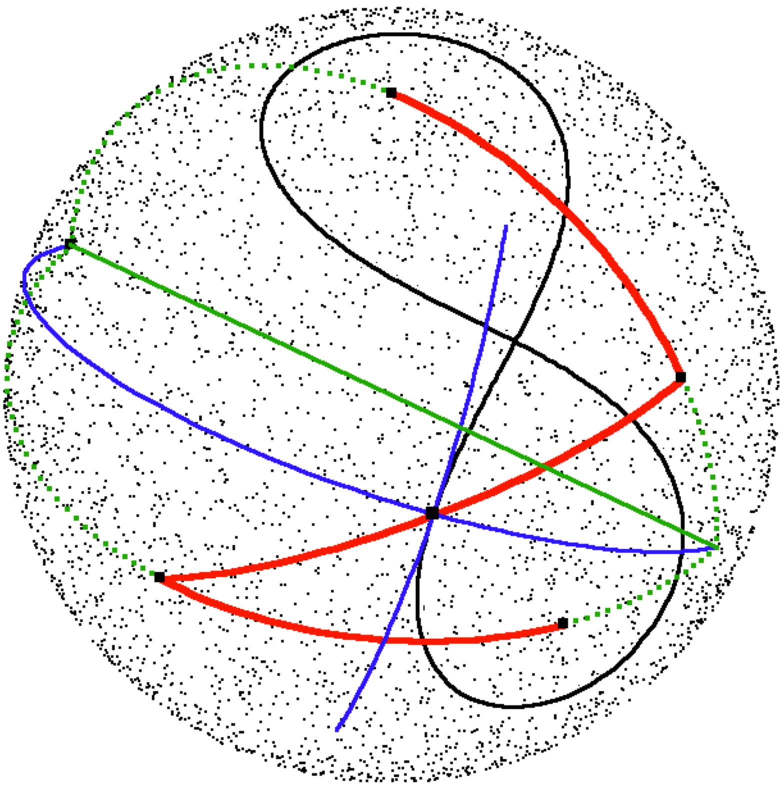
About Spherical Lemniscates*

See ATOs for Spherical Ellipse, Planar Lemniscate



The spherical definition is completely analogous to the planar case. The curves are traced by a mechanical drawing mechanism. It consists of three great circle rods, two shorter ones of (spherical) length $\rho = cc$ and a longer one of length $\lambda = 2 \cdot dd$ (drawn red in the figure). The shorter ones have one endpoint each at 'focal' points F_1, F_2, around which they can rotate. The longer great circle rod connects the two shorter ones, thus creating a mechanism with one degree of freedom: If one of the short rods rotates with con-

* This file is from the 3D-XplorMath project. Please see: http://3D-XplorMath.org/

stant angular velocity, then the connecting long rod forces the other short rod to rotate with non-constant angular velocity. – The parameter $ff \in [0,1]$ chooses the drawing pen on the middle rod and this pen draws the curve.

This demo should be seen as another example of how constructions from Euclidean geometry can be repeated in spherical geometry. See also `About Mechanically Generated Curves` from the Documentation Menu of Planar Curves.

Mechanically generated curves come together with a *construction of their tangents!* Our drawing mechanism is anchored with its focal points on a fixed sphere. On this fixed sphere the drawing takes place. A second *moving sphere* (same radius and midpoint as the fixed sphere) is attached to the drawing part of the mechanical apparatus, in the present case attached to the middle rod. One can imagine that *any point of the moving sphere traces a curve on the fixed sphere!* The velocity vectors of these traced curves give a time-dependent vector field on the fixed sphere. It is a marvellous theorem that *for each fixed time t the vectors of the time-dependent field are the velocity vectors of a standard rotation of the sphere.* In other words: For each fixed time t are the integral curves of this momentary velocity field *concentric circles,* the orbits of a standard rotation. The antipodal centers of these circles are therefore called momentary centers of rotation. Any point $c(t)$ on one of the traced curves can be connected (by a great circle) to the antipodal momentary centers of rotation (drawn blue) and the tangent of the traced curve at $c(t)$ (also blue) is orthogonal to this momentary radius.

How can one find, for a specific drawing mechanism, these momentary centers of rotation? Consider the endpoints of the middle rod of the present mechanism. They are points of the moving sphere. Since they are also the endpoints of the two rotating rods, they can only move orthogonally to their rods. The two rotating rods are therefore always pointing to the momentary center of rotation. We can find these centers by intersecting the two great circles on which the short rods lie (drawn dotted green).

H.K.