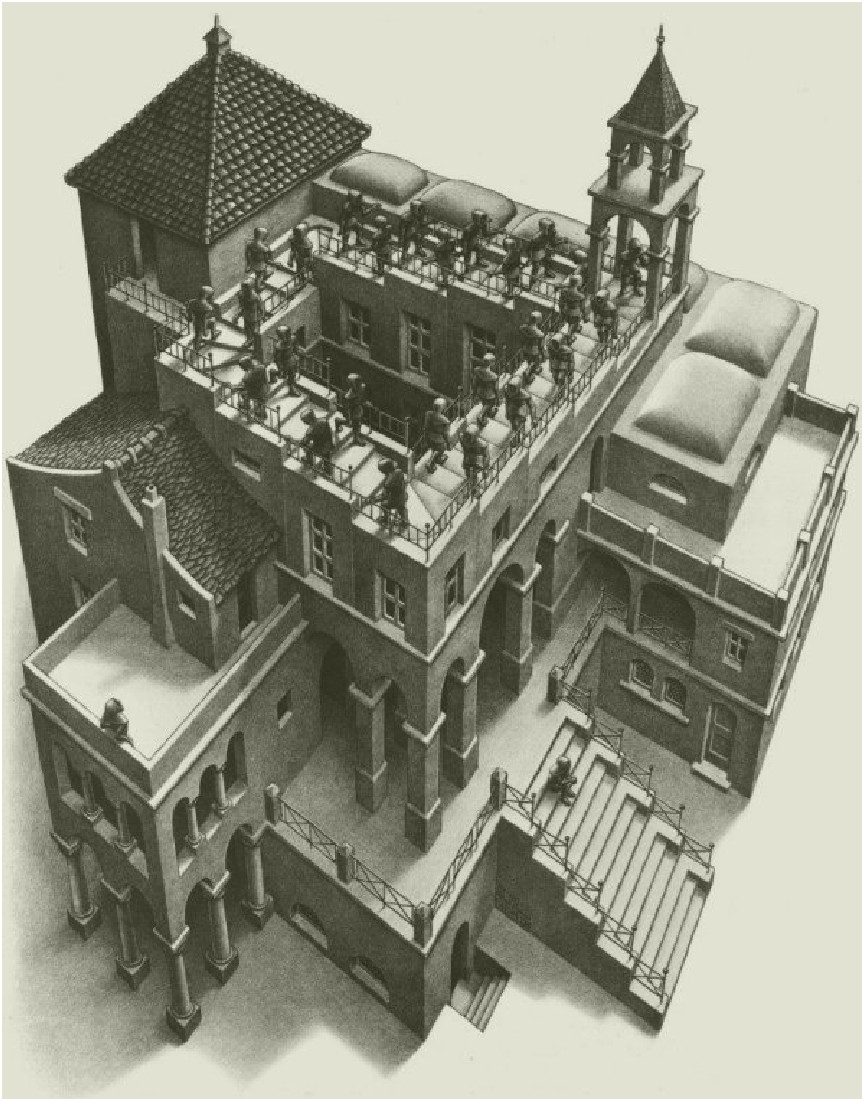
Shepard Tones



Ascending and Descending by M. Escher

If you misread the name of this exhibit, you may have expected to hear some pretty music played on a set of shepherd's pipes. Instead what you heard was a sequence of notes that probably sounded rather unmusical, and may have at first appeared to keep on rising indefinitely, "one note at a time". But soon you must have noticed that it was getting nowhere, and eventually you no-doubt realized that it was even "cyclic", i.e., after a full scale of twelve notes had been played, the sound was right back to where it started from! In some ways this musical paradox is an audible analog of Escher's famous Ascending and Descending drawing.

While this strange "ever-rising note" had precursors, in the form in which it occurs in 3D-XplorMath, it was first described by the psychologist Roger N. Shepard in a paper titled *Circularity in Judgements of Relative Pitch* published in 1964 in the Journal of the Acoustical Society of America.

To understand the basis for this auditory illusion, it helps to look at the sonogram that is shown while the Shepard Tones are playing. What you are seeing is a graph in which the horizontal axis represents frequency (in Hertz) and the vertical axis the intensity at which a sound at a given frequency is played. Note that the Gaussian or bell curve in this diagram shows the intensity envelope at which all sounds of a Shepard tone are played. A single Shepard tone consists of the same "note" played in seven different octaves—namely the octave containing A above middle C (the center of our bell curve) and three octaves up and three octaves down. Thus the first or "A" Shepard tone consists of simultaneously playing the following frequencies (in Hertz): 55, 110, 220, 440, 880 1760, 3520, with each frequency being played at the intensity given by the height of the Gaussian at that frequency. Of course, what our ear hears it interprets as the note A of the scale—not the pure tone of A in some particular octave, but rather a rich harmonic mixture of the A notes from seven adjacent octaves of which the central one, A above middle C, sounds most strongly, while the others get less intense as they go up or down from there.

To get the next (or “B”) Shepard tone, each note of the above mixture is increased by a “half-tone”, that is, its frequency is multiplied by the twelfth root of two. And of course the same bell curve again gives the intensity of each note of the mixture. Now our ear hears the note B (again as a rich harmonic mixture) and by our long experience with the scale, it seems that we have now gone “up” one half-tone from the first note.

Well, you can take it from there. To get the next (third) in the sequence of Shepard Tones, we again multiply every frequency of the B tone by twelfth root of two (i.e., “ascend a half-tone”) and play each tone at the appropriate intensity for that frequency, as given by the bell curve. Again it feels to us that this third Shepard tone is a half-tone “higher” than the second one. And so on for the next nine Shepard tones.

But then as you will see we are right back to where we started!