About the Plane Curves Category

A plane curve is described by a map of an interval, $[tMin, tMax]$ in t-space into (x, y)-space. Each point t is mapped to a two-dimensional point P with coordinates $(x(t), y(t))$. The mapping is given by two real-valued functions that give the values of x and y as a function of the t and nine global "parameters", called $aa, bb, cc, dd, ee, ff, gg, hh, ii$. Most curves use only 1,2,3 or even none of these parameters. (For example, an ellipse depends on two semi-axes, aa and bb). To discretize the curve, the interval $[tMin, tMax]$ is subdivided into $tResolution$ equispaced points running from $tMin$ to $tMax$. These points divide the domain interval into subintervals. When a curve is "Create"-ed, the mapping $t \mapsto (x, y)$ is applied to each of the subdivision points, so each of the subintervals in t-space is mapped to a segment in the plane, and this collection of segments is a polygon that is what actually represents the immersed curve.

When you have selected a particular curve from the Plane Curve menu, a version with certain default parameter values will be displayed. You can then choose "About This Object..." from the Action Menu to see the equations for the curve as a function of t and the parameters, (and perhaps to see some interesting properties of the curve). You can change the parameters in the Settings Menu and then re-Create the curve.

The program can also interpolate linearly between two curves of the same family that you can set by choosing "Set Morphing..." in the Settings Menu. The number of steps in the "morph" is the Number of Frames in the filmstrip, an integer that you can also set. Playing back the filmstrip gives a "movie" of the curve changing gradually ("morphing") between the initial and final curves.

A user can define a curve by choosing one of the User Defined... items from the Plane Curve menu. This will bring up a dialog that will permit one to create algebraic expressions describing the curve involving the variables t and the nine parameters, $aa, bb, ..., ii$. Note that if you want to create the graph of a function, $f(x)$, i.e., display the curve $y = f(x)$, then you can use the parametric equations $x(t) := t, y(t) := f(t)$.

The user can also define a plane curve by giving its curvature, k, as a function of t (and the usual parameters $aa, ...ii$). To do this choose User (Curvature)... from the Planar Curves menu. The resulting curve will have t as its arclength parameter, and will start (at $t = 0$) from the origin with tangent the unit vector in the x direction. (By the fundamental theorem of plane curves, there is a unique such curve.) The interval $[tMin, tMax]$ must contain zero of course—or the curve will be empty.

See the corresponding discussion in About the Surface Category for more detail on how to enter expressions.

Plane curves can also be given "implicitly", as the solutions of an equation, f(x,y) = constant. There are three curves in the Plane Curve menu that are given this way, the Cuspidal Cubic, $y^2 - x^3 = ff$, the Nodal Cubic, $y^2 - x^2 * (1 - x) = ff$, and the Tacnodal Quartic, $y^3 + y^2 - x^4 = ff$. There is also a User (Implicit)... menu item in the Plane Curve menu, that brings up a dialog that will let you enter an expression (defining a function $f(x, y)$) a value for ff, and a rectangle in the x, y-plane. Clicking on the Create button will display the solution of $f(x, y) = ff$ inside the rectangle.

There are several items in the Plane Curve menu that create special animations:

First there is Show Parallel Curves. This first draws the normals to the selected plane curve, from the curve to the center of the osculating circle. It then leaves behind a trace of the evolute (the locus of all centers of osculating circles) and draws the parallel curves to the selected curve. (Singularities develop when the parallel curve reaches the evolute.)

Secondly, there is Draw Generalized Cycloid. This rolls a circle (of radius hh) on the selected curve, and a point D along a fixed radius traces out a curve. (If $ii = 0$ the drawing point, D, is the center of the circle, if $ii = 1$, it is the point on the rim, and in general ii is the signed distance from the center in units of the radius. Changing the sign of hh will change the side of the curve on which the circle rolls. If gg is not zero, then the value of hh (the radius) is modified to the nearest value that makes the length of the circle go an integral number of times into the length of the curve (so that the cycloid closes up). If you change the parameters gg, hh, ii, and then choose Draw Generalized Cycloid again, then a new generalized cycloid is drawn of course, and the old one is first erased unless you hold down Shift as you make the menu selection.

Finally, selecting Draw Osculating Circles will draw (in blue) the osculating circles of a selected parametric curve (and their radii) at a point that moves along the curve. As the point moves, the centers of the osculating circles trace out the evolute of the curve in red. (If you press the Shift key while selecting Draw Osculating Circles, the normals to the radii of the osculating circles will be drawn in red before the osculating circles are drawn.)