The Nephroid *

The Nephroid is generated by rolling a circle of one radius on the outside of a second circle of twice the radius. In 3D-XplorMath, either choose `Nephroid` from the Plane Curve menu, or choose `Circle`, then select `Set Parameters...` from the Settings menu and set $hh = -0.5 \cdot aa$, $ii = 1$. With $R = 3r$ we thus have the parametrization for Nephroids:
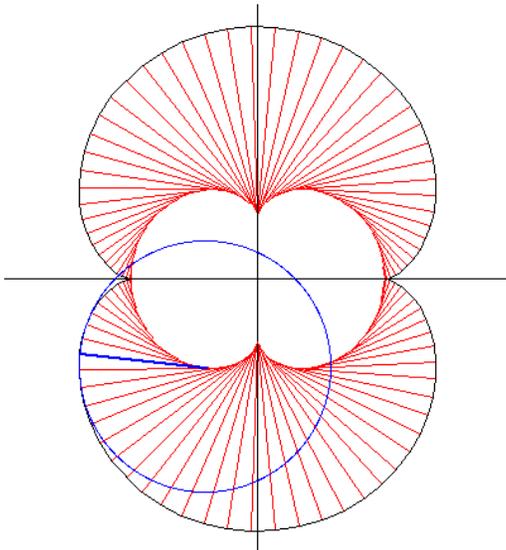
$$x(t) = R \cdot \sin(t) + r \cdot \sin(3t)$$
$$y(t) = R \cdot \cos(t) + r \cdot \cos(3t)$$

As with Cardioids and Limaçons, one can also make the radius for the drawing stick shorter or longer: Set $ii > 1$ for the looping relatives of the Nephroid or see the default `Morph` in the Animation menu.

The complex map $z \mapsto z^3 + 3z$ maps the unit circle to such a Nephroid. To see this, in the Conformal Map Category, select $z \mapsto z^e e + eez$ from the Conformal Map menu, and then choose Set Parameters from the Settings menu and set *ee* to 3.

The normals of one Nephroid have as envelope another, smaller Nephroid—the same phenomenon as for the Cardioid and the Cycloid. (To see this select `Show Osculating Circles With Normals` from the Action menu). In technical jargon, the caustics for each of these curves is a similar curve.

H.K.