

Loxodrome*

A Loxodrome (also called a rhumb line) is a route that a boat would take if it kept a constant compass heading (so that on a Mercator projection it is simply a straight line). To be more formal, a loxodrome is a path that lies on the unit sphere in \mathbb{R}^3 and that makes a constant angle with the great circles of longitude (i.e. the meridians). Thus the loxodromes are analogous to the logarithmic spirals in the (complex) plane, which make a constant angle with the rays through the origin. In fact, since stereographic projection from the complex plane to the unit sphere is conformal (in other words: angle-preserving) and since the stereographic projection of the radial lines in the plane are the circles of longitude, it follows that the loxodromes are given by stereographically projecting the logarithmic spirals. On the other hand, since the exponential map of the complex plane to itself is conformal and maps the lines parallel to the real axis to radial lines, it follows that the logarithmic spirals are just the images under the exponential map of straight lines, i.e. the images of $t \mapsto (aa + i) \cdot t$. Hence we can define the loxodromes parametrically by

$$t \mapsto \text{StereographicProjection}(\exp((aa + i) \cdot t)).$$

Note that the osculating circles all lie on the sphere.

R.S.P.

* This file is from the 3D-XplorMath project. Please see:

<http://3D-XplorMath.org/>