Algebraic Functions with Singularities in 3DXM*

<u>*CayleyCubic*</u> :

$f(x, y, z) := 4(x^2 + y^2 + z^2) + 16\,x\,y\,z - 1, \quad ff = 0.$

This cubic has 4 cone singularities at the vertices of a tetrahedron. The other surfaces in the `Default` ff`-Morph` are nonsingular.

<u>*ClebschCubic*</u> :

$f(x, y, z) :=$
$81(x^3 + y^3 + z^3) - 189(x^2(y + z) + y^2(z + x) + z^2(x + y)) + 54xyz + 126(xy + yz + zx) - 9(x^2 + x + y^2 + y + z^2 + z) + 1.$

This cubic has no singularities but is famous for the 27 lines that lie on it. The lines are shown in 3DXM. The surface has tetrahedral symmetry.

<u>*DoublyPinchedCubic*</u> :

$f(x, y, z) := z(x^2 + y^2) - x^2 + y^2.$

This cubic has two pinch-point singularities at ± 1 on the z-axis. The segment between the singularities lies on it. The whole z-axis satisfies the equation; the `Default Morph` shows how an infinite spike converges to this line.

<u>*KummerQuartic*</u> :

$\lambda := (3aa^2 - 1)/(3 - aa^2),$

$$f(x, y, z) := (x^2 + y^2 + z^2 - aa^2)^2 \\ \qquad - \lambda((1 - z)^2 - 2x^2)((1 + z)^2 - 2y^2), \quad aa = 1.3.$$

This quartic has 4+12 cone singularities and tetrahedral symmetry. Six noncompact pieces, each with two cone

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* This file is from the 3D-XplorMath project. Please see:

http://3D-XplorMath.org/

points, are connected by five compact pieces which look like curved tetrahedra. The singularities survive small changes, see the `Default Morph` : $1.05 \leq aa \leq 1.5, ff = 0$.

BarthSextic :
$c_1 := (3+\sqrt{5})/2,\ c_2 := 2+\sqrt{5}$
$f(x,y,z) :=$
$4(c_1x^2 - y^2)(c_1y^2 - z^2)(c_1z^2 - x^2) - c_2(x^2+y^2+z^2-1)^2.$
Barth's Sextic has icosahedral symmetry. 20 tetrahedron-like compact pieces are placed over the vertices of a dodecahedron so that each tetrahedron has 3 of its vertices at midpoints of dodecahedron edges. This accounts for 30 of the cone singularities. Each of the 20 outward pointing vertices of the tetrahedra is connected via a cone singularity to a cone-like noncompact piece of the Sextic. The `Default Morph` embeds this singular surface in a family of nonsingular sextics. `Use Raytrace Rendering`.

D4 :
$f(x,y,z) := 4x^3 + (aa - 3x)(x^2+y^2) + bbz^2$
This family of cubics has a $D4$-singularity. At $bb = 0$ the family degenerates into three planes, intersecting along the z-axis.

UserDefined: Our example is the *Cayley Cubic*, see above.

H.K.