The Fujimori-Weber Surfaces

S. Fujimori and M. Weber derived in [FW] a Weierstrass representation for a large collection of embedded triply periodic minimal surfaces. In 3D-XplorMath ten of their families are realized, choose $ff = 1, \ldots, 10$. For each ff we get a 1-parameter family, where the parameter cc controls the length ratio between the horizontal and vertical closed symmetry lines, see the default morph.

In the third part of `About Minimal Surfaces` (available from the Documentation Menu) we explain a construction of these surfaces with the help of minimal surface pieces which are bounded by hexagons in $\mathbb{R}^3$. To see these fundamental pieces, select in the Action Menu `Don't Show Reflections` and look also at the conjugate piece (which is the one bounded by a hexagon). In Wire Frame and in Point Cloud Display one can emphasize the boundary in the Action Menu.

Some of these surfaces can also be seen with a different parametrization in 3D-XplorMath. The parameter lines on the Fujimori-Weber surfaces are level lines and lines of steepest descent for the height function in z-direction. For the other triply periodic surfaces the level lines are pull backs of polar coordinates under a complex differentiable function defined on the minimal surface.

The Weber-Fujimori surfaces give for
$ff = 2$ the Schwarz P-surface,
$ff = 5$ the Schwarz H-surface,
$ff = 8$ the A Schoen S-S-surface,

$ff = 7$ the same surface *'inside-out'*.
For minimal surfaces which carry straight lines there is no geometric distinction between the two sides, because 180° rotation around such a line maps the minimal surface onto itsself, but interchanging the two sides. If there are no lines on the surface then the two sides may look so different that they appear to be different surfaces. But in reality the fundamental pieces are only assembled in different ways – showing mainly one side of the surface in one case and the other side in the other case (choose in the View Menu: `Distinguish Sides By Color`). The other such pairs are
$ff = 1, 3$ (A. Schoen's H-T-surface, also in 3DXM),
$ff = 4, 6$ (A. Schoen's H-R-surface),
$ff = 9, 10$ (A. Schoen's T-R-surface).

[FW] Fujimori S., Weber M.: Triply Periodic Minimal Surfaces Bounded By Vertical Symmetry Planes. Manuscripta Math. 129, 29 - 53 (2009).

H.K.