

Dirac Belt and Feynman Plate Tricks *

Dirac invented the famous belt trick to demonstrate a property of the motions of Euclidean Space that is indeed difficult to imagine without visual help.

The trick is performed with a strip, or belt, that is initially parallel to the screen. The orthogonal projection of the performance looks as follows: The left end of the belt stays fixed, the right end moves around the left end in a circular motion. It is important that the moving end **stays parallel** to the fixed end through the whole trick (parallel means: the final edge and the final normal each stay parallel to the initial edge and initial normal). One observes with surprise:

After moving the right end once around the circle the belt is twisted twice. After the second circular move the belt is untwisted (as it was initially).

The trick is shown in stereo because it is impossible that the belt stays in its initial plane when the ends are moved as described. It is important to visualize how the different parts of the belt move **vertically** to the screen.

It will be no surprise to observe that the first circular movement – when looked at in 3D – is different from the second circular movement. During the first circular movement the

* This file is from the 3D-XplorMath project. Please see:

<http://3D-XplorMath.org/>

middle part of the belt moves vertically to be **in front** of the screen while the two ends stay on the screen. During the second circular movement it is the other way round: the middle part of the belt moves vertically to be **behind** the screen while the ends continue to be at their fixed vertical position. As soon as one can fix this image in one's mind it is obvious how the circular motion with parallel ends produces the twist of the belt.

Another instance of the same property of the Group of Euclidean motions is the **Feynman Plate Trick** or *Waiter's Cup Trick* : It is possible to continuously rotate a cup on ones horizontal hand in the **same** direction if during the first rotation the hand is above elbow height, during the second rotation below elbow height and so on, alternatingly above and below elbow height. Namely, imagine that the shoulder is the fixed end of the belt and the always horizontal – but continuously rotating – middle part of the belt is the hand. Choose **Do Plate Trick** from the Animation Menu to see half the belt performing the trick.

Some people cannot believe what they see. In such a case one can switch to **Monocular Vision** and **Orthographic Projection** in the View Menu to watch the belt – without at all observing the motion vertical to the screen. Or try **Patch Display** for a solid belt.

B.P.