# About the Costa-Hoffman-Meeks Minimal Surfaces

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The original Costa surface was responsible for the rekindling of interest in minimal surfaces in 1982. It is a minimal **embedding** of the 3-punctured square torus. Its planar symmetry lines cut this surface into four conformal squares and the two straight lines through the saddle are the diagonals of these squares. Because of the emphasis on the symmetries, our formulas are taken from [K2.] The Costa-Hoffman-Meeks surfaces are generalizations of the Costa surface; their genus grows as the dihedral symmetry (controlled by dd) is increased. The underlying Riemann surfaces are tesselated by hyperbolic squares with angles $\frac{\pi}{k}$, $(k = 2, 3, ...)$.

The Gauss map of such a surface is determined by its qualitative properties only up to a multiplicative factor cc which we suggest for the morphing (as in the Chen-Gackstatter case). It closes the period (at cc0) by an intermediate value argument.

As in Costa's case, the qualitative picture determines the Gauss map only up to a multiplicative factor. The standard morph shows the dependence of the surfaces on this factor, closing the period at cc0.

[Hoffman, Karcher] Complete embedded minimal surfaces of finite total curvature. Encyclopaedia of Mathematical Sciences, vol. 90. Geometry V (Ed. R. Osserman), pp. 5-93

[K2] H. Karcher, Construction of minimal surfaces, in "Surveys in Geometry", Univ. of Tokyo, 1989, and Lecture Notes No. 12, SFB 256, Bonn, 1989, pp. 1–96.

For a discussion of techniques for creating minimal surfaces with various qualitative features by appropriate choices of Weierstrass data, see either [KWH], or pages 192–217 of [DHKW].

[KWH] H. Karcher, F. Wei, and D. Hoffman, The genus one helicoid, and the minimal surfaces that led to its discovery, in "Global Analysis in

Modern Mathematics, A Symposium in Honor of Richard Palais' Sixtieth Birthday", K. Uhlenbeck Editor, Publish or Perish Press, 1993

[DHKW] U. Dierkes, S. Hildebrand, A. Kuster, O. Wohlrab, Minimal Surfaces I, Grundlehren der math. Wiss. v. 295 Springer-Verlag, 1991