

About the Schwarz PD Family

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This is a 2-parameter family of triply periodic genus 3 surfaces. In each case the original surface and the conjugate surface are embedded. The most symmetric example (with a cubic lattice) which is obtained when $cc = 0$, $dd = 0$, was already constructed by H. A. Schwarz. When Alan Schoen found more triply periodic surfaces around 1970 he named the two surfaces which Schwarz found the P-surface (P for cubic primitive) and the D-surface (D for diamond). He also found a third embedded(!) surface in the associate family of these, the Gyroid (associate parameter 0.577 which is approx. 52 degrees). If $dd=0$ then a fundamental cell for the lattice is a prism with square base. In the morphing cc changes the height of the prism.

K. Grosse-Brauckmann, M. Wohlgemuth: The Gyroid is embedded and has constant mean curvature companions. To appear Calc. Var. 1996

For a discussion of techniques for creating minimal surfaces with various qualitative features by appropriate choices of Weierstrass data, see either [KWH], or pages 192–217 of [DHKW].

[KWH] H. Karcher, F. Wei, and D. Hoffman, The genus one helicoid, and the minimal surfaces that led to its discovery, in “Global Analysis in Modern Mathematics, A Symposium in Honor of Richard Palais’ Sixtieth Birthday”, K. Uhlenbeck Editor, Publish or Perish Press, 1993

[DHKW] U. Dierkes, S. Hildebrand, A. Kuster, and O. Wohlrab, Minimal Surfaces I, Grundlehren der math. Wiss. v. 295 Springer-Verlag, 1991