

About the Schwarz H family

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This is a 1-parameter family of triply periodic surfaces. The surfaces are made of pieces which one could call “triangular catenoids”; annular Plateau solutions bounded by two parallel equilateral triangles. In the morphing aa changes the height-to-edge length ratio of these triangular catenoids. Observe that, as in the case of circles bounding catenoids, the distance between the triangles has to be small enough and then they bound a stable and an unstable triangular catenoid. In the PD-family with $dd=0$ one can observe analogous “square catenoids”, except that our parametrization does not emphasize them.

When Alan Schoen found more triply periodic surfaces around 1970 he named these “Schwarz’ H surfaces”. (Maybe Schwarz constructed only one member of the family.) Later, the Swedish chemist, Lidin, found another embedded example (now called the Lidinoid) in the same associate family, when aa is approximately 0.55, and the associate family parameter is 0.7139, (about 64.25 degrees).

For a discussion of techniques for creating minimal surfaces with various qualitative features by appropriate choices of Weierstrass data, see either [KWH], or pages 192–217 of [DHKW].

[KWH] H. Karcher, F. Wei, and D. Hoffman, The genus one helicoid, and the minimal surfaces that led to its discovery, in “Global Analysis in Modern Mathematics, A Symposium in Honor of Richard Palais’ Sixtieth Birthday”, K. Uhlenbeck Editor, Publish or Perish Press, 1993

[DHKW] U. Dierkes, S. Hildebrand, A. Kuster, and O. Wohlrab, Minimal Surfaces I, Grundlehren der math. Wiss. v. 295 Springer-Verlag, 1991