About the Lopez-Ros No-Go Theorem

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The theorem of Lopez-Ros [LR] says that a complete, minimal embedding of a punctured sphere is either a catenoid or a plane.

Our example is parametrized by a 3-punctured sphere, and its Gauss map is given by $\text{Gauss}(z) := \text{cc}(z-1)(z+1)$. Parameter lines on the sphere extend polar coordinates around the punctures at $z = +ee, z = -ee, z = \infty$.

A necessary condition for embeddedness is parallel normals at infinity, i.e., $ee = 1$. In this case the period cannot be closed. If $ee \neq 1$, then cc can be chosen to close the period, but then the catenoid ends are tilted so that they intersect the third (planar) end. – For each $ee \neq 1$ we set cc0 to the value which closes the period; one can therefore see in the morphing how cc is used to close the period.

[LR] F.J. Lopez and A. Ros, On embedded complete minimal surfaces of genus zero, Journal of Differential Geometry 33 (1), 1991, pp 293–300

For a discussion of techniques for creating minimal surfaces with various qualitative features by appropriate choices of Weierstrass data, see either [KWH], or pages 192–217 of [DHKW].

[KWH] H. Karcher, F. Wei, and D. Hoffman, The genus one helicoid, and the minimal surfaces that led to its discovery, in "Global Analysis in Modern Mathematics, A Symposium in Honor of Richard Palais' Sixtieth Birthday", K. Uhlenbeck Editor, Publish or Perish Press, 1993

[DHKW] U. Dierkes, S. Hildebrand, A. Kuster, and O. Wohlrab, Minimal Surfaces I, Grundlehren der math. Wiss. v. 295 Springer-Verlag, 1991