The Viviani Curve *

The Viviani curve is the intersection of a sphere of radius $2 \cdot aa$ and a cylinder of radius aa that touch at a single point, the double point of the curve. Parametric formulas for it are:

$$z = aa\ (1 + \cos(t)) = aa\ 2\ \cos(t/2)^2,$$

$$y = aa\ \sin(t) = aa\ 2\ \sin(t/2)\ \cos(t/2), \text{ and}$$

$$x = aa\ 2\ \sin(t/2)$$

Implicit equations for the two intersecting surfaces are:

$$x^2 + y^2 + z^2 = 4\ aa^2, \quad \text{a sphere of radius } 2\ aa,$$

$$(z - aa - bb)^2 + y^2 = aa^2, \quad \text{a cylinder of radius } aa.$$

The planar projections of this curve are therefore in general curves of degree 4, but because of its symmetries the Viviani curve has two orthogonal two-to-one projections that are simpler; namely curves of degree 2. Indeed projecting it to the y-z-plane we get a twice covered circle (use Settings Menu: Set Viewpoint and Up Direction 200,0,0), projecting to the x-z-plane gives a twice covered parabolic piece, $(1 - z/(2aa)) = (x/(2aa))^2$, while the projection to the x-y-plane is the degree 4 figure 8 with the equation (for $aa = 1/2$): $x^2 - y^2 = x^4$.
Note that the osculating circles lie on the sphere.

R.S.P.

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* This file is from the 3D-XplorMath project. Please see: http://3D-XplorMath.org/