About the Saddle Tower Surface

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These examples generalize Scherk's conjugate pair of singly-periodic/doubly-periodic minimal surfaces. The singly-periodic examples stay embedded if the dihedral symmetry (and with it the number of punctures) is increased ($\text{Gauss}(z) = z^k, k = ee-1$). The most symmetric ones (bb=0.5/ee) can be deformed by decreasing bb. See [K1], [K2] for more details.

These surfaces are parametrized by punctured spheres, but the Weierstrass integrals have periods, a vertical one in the singly periodic case, two horizontal ones for doubly periodic surfaces. The parameter lines extend polar coordinates around the punctures to the whole sphere—in these cases giving level lines on the surfaces.

The degree of dihedral symmetry is, of course, a discrete property, and it is controlled by the parameter ee. Thus, ee should be set to an integer (the default is 2). For each choice of ee, changing bb gives a one-parameter family of surfaces, of which the most symmetric member is obtained by setting bb = 0.5/ee. Try setting ee to 3 and 4, and bb to 0.333 and 0.25 respectively. The wings of the singly periodic saddle tower surfaces become parallel in pairs if (for $ee > 2$) one sets bb = 0.0825. These stay embedded for ee=3 and ee=4.

We also recommend viewing the associate family morphing.

[K1] H. Karcher, Embedded minimal surfaces derived from Scherk's examples, Manuscripta Math. 62 (1988) pp. 83–114.

[K2] H. Karcher, Construction of minimal surfaces, in "Surveys in Geometry", Univ. of Tokyo, 1989, and Lecture Notes No. 12, SFB 256, Bonn, 1989, pp. 1–96.

For a discussion of techniques for creating minimal surfaces with various qualitative features by appropriate choices of Weierstrass data, see either [KWH], or pages 192–217 of [DHKW].

[KWH] H. Karcher, F. Wei, and D. Hoffman, The genus one helicoid, and the minimal surfaces that led to its discovery, in "Global Analysis in Modern Mathematics, A Symposium in Honor of Richard Palais' Sixtieth Birthday", K. Uhlenbeck Editor, Publish or Perish Press, 1993

[DHKW] U. Dierkes, S. Hildebrand, A. Kuster, and O. Wohlrab, Minimal Surfaces I, Grundlehren der math. Wiss. v. 295 Springer-Verlag, 1991