Lissajous, Double Helix, Column, Norm 1 Family *

The French mathematician Jules Antoine Lissajous (1882-1880) studied vibrating objects by reflecting a spot of light of them, so that the various modes of vibration gave rise to *Lissajous curves*, see `Plane Curve Category`. Lissajous Space Curves and **Lissajous Surfaces** are a natural mathematical generalization. We use the

$$\textit{Parametrization: } F(u,v) = \begin{pmatrix} \sin u \\ \sin v \\ \sin((dd - aa\,u - bb\,v)/cc) \end{pmatrix}.$$

The `Default Morph` joins a surface with tetrahedral symmetry and conical singularities and a surface with cubical symmetry and 12 pinch point singularities.

The **Double Helix** is a reminder of the famous double helix from genetics. For playing purposes there are two more parameters with default values $dd = 0, ee = 0$. We use the *parametrization*:

$$AA := aa + dd\,u,\ \alpha := (1 - ee\,u)u,$$

$$F(u,v) = \begin{pmatrix} AA((1-v)\cos\alpha + v\cos(\alpha + bb\,\pi)) \\ AA((1-v)\sin\alpha + v\sin(\alpha + bb\,\pi)) \\ cc\,u - 3.5 \end{pmatrix}.$$

This is a family of *ruled surfaces*: try the `Default Morph,` it varies the limits of the parameter v.With $bb = 1$ we get the Helicoid. The default parameters have been taken from

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* This file is from the 3D-XplorMath project. Please see:

http://3D-XplorMath.org/

Watson-Crick and give a reasonably good representation of molecular DNA. Dick Palais' biologist friend Chandler Fulton suggested the example and helped to get it right, many thanks.
We suggest to select `Move Principal Curvature Circles` from the Action Menu; this is seen best in `Point Cloud Display` from the View Menu.

Column Surface is used here in the sense of an architectural column, see the article by Marty Golubitzky and Ian Melbourne at

http://www.mi.sanu.ac.rs/vismath/golub/index.html

The order of the rotational symmetry around the z-axis is chosen with the 3DXM-parameter ii. Additional symmetry types can be selected with the parameter $hh = 1, \ldots, 6$; for other values of hh the unsymmetrized column shape is given by a formula that depends on the parameters $aa, \ldots, gg, ii$ and on the coordinates (θ, z). The formula is not determined by geometric properties, but is intended for playing.
The `Default Morph`, with $hh = 0$, varies the shape only mildly.

The **Norm 1 Family** is defined by the *implicit equation:*

$$f(x, y, z) = (|x|^p + |y|^p + |z|^p)^{1/p} = 1, \qquad 0 < p < \infty.$$

We get at $p = 1$ an *Octahedron*, at $p = 2$ a *Sphere* and at $p = \infty$ a *Cube*. For $1 \leq p \leq \infty$ these surfaces can be

viewed as the unit sphere in $\mathbb{R}^3$ for a Banach metric determined by p.
We *parametrize* these surfaces by spherical polar coordinates:

$$xp := \sin v \cos u,\ \ yp := \sin v \sin u,\ \ zp := \cos v,$$
$$x := \text{sign}\,(xp)|xp|^e,\ \ y := \text{sign}\,(yp)|yp|^e,\ \ z := \text{sign}\,(zp)|zp|^e.$$

To obtain a reasonable family we set the exponent e in terms of the `default morphing` parameter ee as follows:

$$e := 1 + \tan(ee), \quad -\pi/4 < ee < \pi/2.$$

We obtain the Sphere at $ee = 0$,
the Octahedron at $ee = \pi/4$.
Numerical reasons prevent computation at $ee = -\pi/4$ (anyway a degenerate surface) and at $ee = \pi/2$, the Cube.
Already where we stop the computation the Pascal-values of sin near π had to be improved.