About Planar Enneper

H. Karcher

The surfaces Wavy Enneper, Catenoid Enneper, Planar Enneper, and Double Enneper are finite total curvature minimal immersions of the once or twice punctured sphere—shown with standard polar coordinates. These surfaces illustrate how the different types of ends can be combined in a simple way.

The pure Enneper surfaces ($aa = 0,\ dd = 0$) and the Planar Enneper surfaces ($aa = 0, dd = 2$) with Weierstrass data:

Gauss map : $Gauss(z) = z^{ee+1}(1 + aa\, z^{ff})$
Differential: $dh = scaling \cdot Gauss(z)/z^{dd}\, dz$

have been re-discovered many times, because the members of the associate family are *congruent* surfaces (as can be seen in the interesting associate family morphing!!) and the Weierstrass integrals integrate to polynomial (respectively rational) immersions. For $aa = 0,\ dd = 1$ one does not obtain a finite total curvature surface, but a periodic surface that looks like a half-plane with periodically attached Enneper pieces. One obtains larger pieces of this *Wavy Plane* if one either increases ee or the range of v. The members of the associate family are also congruent, try *Cyclic Associate Family Morph*.

For small $aa \neq 0$ one has wavy perturbations of the Enneper end.

For the ($aa = 0$)-examples ee is an integer valued parameter that determines the degree of dihedral symmetry of the surfaces. Morphing the range of u helps to imagine these surfaces by starting with a plane minus a disk and then observe how an Enneper end is attached. In the Set Morphing Dialog first press *Initiate to current parameters*, then choose $umax0 = -0.5$, $umax1 = 0.6$.

Formulas are taken from:

H. Karcher, Construction of minimal surfaces, in "Surveys in Geometry", Univ. of Tokyo, 1989, and Lecture Notes No. 12, SFB 256, Bonn, 1989, pp. 1–96.

For a discussion of techniques for creating minimal surfaces with various qualitative features by appropriate choices of Weierstrass data, see either [KWH], or pages 192–217 of [DHKW].

[KWH] H. Karcher, F. Wei, and D. Hoffman, The genus one helicoid, and the minimal surfaces that led to its discovery, in "Global Analysis in Modern Mathematics, A Symposium in Honor of Richard Palais' Sixtieth Birthday", K. Uhlenbeck Editor, Publish or Perish Press, 1993

[DHKW] U. Dierkes, S. Hildebrand, A. Kuster, and O. Wohlrab, Minimal Surfaces I, Grundlehren der math. Wiss. v. 295 Springer-Verlag, 1991