Three Pseudospherical Surfaces,*

Dini Family, Kuen, Breather

The theory to obtain these surfaces of Gauss curvature $K = -1$ is explained in

`About Pseudospherical Surfaces.`

The construction begins with explicit solutions of the Sine-Gordon-Equation (SGE). A first and a second fundamental form are written down in terms of an SGE solution. These surface data satisfy the Gauss-Codazzi integrability conditions, hence determine immersed surfaces. For the earliest examples this integration of the surface data could be carried out explicitly. See the quoted text above for more details.

Parametrization of the Dini surfaces:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}(u,v) := \begin{pmatrix} u - t(u,v) \\ r(u,v)\cos(v) \\ r(u,v)\sin(v) \end{pmatrix}, \text{ where}$$

$$\psi := aa \cdot \pi, \; aa \in [0.001, 0.999],$$

$$aa = 0.5 \;\text{ gives the Pseudosphere,}$$

$$g(u,v) := (u - v \cdot \cos(\psi))/\sin(\psi),$$

$$s(u,v) := \exp(g(u,v)),$$

$$r(u,v) := 2\sin(\psi)/(s(u,v) + 1/s(u,v)),$$

$$t(u,v) := 0.5 \cdot r(u,v) \cdot (s(u,v) - 1/s(u,v)).$$

* This file is from the 3D-XplorMath project. Please see: http://3D-XplorMath.org/

Parametrization of the Kuen Surface:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}(u,v) := \begin{pmatrix} \dfrac{2\cosh(v)(\cos(u) + u\cdot\sin(u))}{\cosh(v)\cdot\cosh(v) + u\cdot u} \\ \dfrac{2\cosh(v)(-u\cdot\cos(u) + \sin(u))}{\cosh(v)\cdot\cosh(v) + u\cdot u} \\ v - \dfrac{2\sinh(v)\cosh(v)}{\cosh(v)\cdot\cosh(v) + u\cdot u} \end{pmatrix}$$

Parametrization of the Breather surfaces:

Parameter of the family is $aa \in (0,1)$.
If $w := \sqrt{1 - aa^2}$ is rational then the surfaces are periodic.

$$denom := aa\cdot((w\cosh(aa\cdot u))^2 + (aa\sin(w\cdot v))^2)$$

$$x(u,v) := -u + \frac{2(1 - aa^2)}{denom}\cosh(aa\cdot u)\sin(aa\cdot u)$$

$$y(u,v) := \frac{2w\cosh(aa\,u)}{denom}(-w\cos(v)\cos(w\,v) - \sin(v)\sin(w\,v))$$

$$z(u,v) := \frac{2w\cosh(aa\,u)}{denom}(-w\sin(v)\cos(w\,v) + \cos(v)\sin(w\,v))$$

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