About the Costa Minimal Surface

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This surface was responsible for the re-kindling of interest in minimal surfaces in 1982. It is a minimal **embedding** of the 3-punctured square torus. Its planar symmetry lines cut this surface into four conformal squares and the two straight lines through the saddle are the diagonals of these squares. Because of the emphasis on the symmetries, our formulas are taken from [K2.]

The Gauss map of such a surface is determined by its qualitative properties only up to a multiplicative factor cc which we suggest for the morphing (as in the Chen-Gackstatter case). It closes the period (at cc0) with an intermediate value argument.

After Costa's existence discovery, Hoffman-Meeks proved embeddedness; they also found a deformation family through rectangular tori, where the middle end deforms from a planar one to a catenoid end. They generalized this family to any genus by increasing the dihedral symmetry.

[K2] H. Karcher, Construction of minimal surfaces, in “Surveys in Geometry”, Univ. of Tokyo, 1989, and Lecture Notes No. 12, SFB 256, Bonn, 1989, pp. 1–96.

For a discussion of techniques for creating minimal surfaces with various qualitative features by appropriate choices of Weierstrass data, see either [KWH], or pages 192–217 of [DHKW].

[KWH] H. Karcher, F. Wei, and D. Hoffman, The genus one helicoid, and the minimal surfaces that led to its discovery, in “Global Analysis in Modern Mathematics, A Symposium in Honor of Richard Palais’ Sixtieth Birthday”, K. Uhlenbeck Editor, Publish or Perish Press, 1993

[DHKW] U. Dierkes, S. Hildebrand, A. Kuster, and O. Wohlrab, Minimal Surfaces I, Grundlehren der math. Wiss. v. 295 Springer-Verlag, 1991