Surfaces of revolution with constant Gauß curvature

A surface of revolution is usually described by giving its meridian curve $s \mapsto (r(s), h(s))$. The surface is then obtained by rotation:

$$(x, y, z) := (r \cos \varphi, r \sin \varphi, h).$$

Any kind of curvature condition can be expressed as a differential equation for the meridian curve.

The case of constant Gauß curvature K is particularly simple if the meridian is parametrized by arclength, i.e., $r'^2 + h'^2 = 1$. In this case the meridian is determined by

$$r''(s) + K \cdot r(s) = 0, \qquad h(s) = \int_0^s \sqrt{1 - r'(t)^2} dt.$$

We describe the three kinds of examples in the case $K = 1$.

Sphere:

$$r(s) = \sin(s), \qquad 0 \leq s \leq \pi$$

With cone points:

$$r(s) = a\sin(s), \quad 0 \leq s \leq \pi, \ 0 < a < 1$$

With singularity curve:

$$r(s) = a\sin(s), \quad b \leq s \leq \pi - b, \ 1 < a, \ \cos(b) := 1/a.$$